

Methods of Signal Processing Applied to Gyrokinetic Simulations

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ABSTRACT: Using signal processing techniques, it is possible to make inferences about frequencies, damping rates, and mode structures in shorter runs of data. Transient signals can also be studied using these tools. Two recent signal processing methods are discussed in this paper: damped multiple signal classification and stochastic system identification. Alfvén modes in a tokamak are simulated using this method.

KEYWORDS: Signal Processing, Numerical simulations, Gyrokinetic Simulations, Plasma, plasma simulations.

I. INTRODUCTION

Large computational resources and extended run times are required for numerical simulations, such as those required for plasma physics models such as electromagnetic gyrokinetics. Shorter runs can be used to establish inferences about frequencies, damping rates, and mode structures using signal processing methods. It is also possible to analyse transitory signals using these technologies. Damped multiple signal classification and stochastic system identification are two recent signal processing approaches that we cover in this tutorial. Plasma physics theory relies heavily on initial value codes for gyrokinetic simulations. They are, however, exceedingly time-consuming, especially when dealing with electromagnetic disturbances. Aiming to obtain findings from simulations using the fewest possible time steps is desired. If a system is either damped or unstable, initial-value codes are typically utilised to discover the most prevalent mode. For modes with identical growth or damping rates, simulations must be run until the ratio of mode amplitudes is big enough to distinguish the modes. Transient phenomena such as finding the frequencies of a group of substantially damped modes are also problematic. [1]

Increasingly, it is recognized that fusion power and the lifespan of plasma-facing components are closely linked to the parameters of a tokamak's boundary plasma. Gyrokinetic turbulence models, which have been used to accurately represent turbulence and transport in the core, are required for accurate quantitative modelling and enhanced qualitative knowledge of the border plasma. Developing new gyrokinetic codes or making considerable adjustments to current core gyrokinetic codes is required for boundary-plasma simulation.

Theoretical plasma physics relies heavily on gyrokinetic simulations based on starting value codes. In terms of computation time, they can be prohibitively expensive, particularly for electromagnetic perturbations. Aiming to obtain findings from simulations using the fewest possible time steps is desired. If a system is either damped or unstable, initial-value codes are typically utilised to discover the most prevalent mode. Simulations must be run long enough so that the ratio of the mode amplitudes may be clearly discerned if the system comprises modes with identical growth or damping rates. When dealing with transitory events, such as finding the frequencies of a group of heavily damped modes, a second issue surfaces. The frequency resolution Δf is inversely proportional to the signal length in traditional Fourier transform (FT) methods. As a result, long-term traces are needed to differentiate modes with identical frequencies or to determine damping rates. In any case, the modes of interest may have degenerated to such an extent that the signal is no longer usable due to, for example, numerical noise corruption (which especially can be a problem for Monte Carlo simulations). In addition, a longer time trail may not provide higher resolution if the phenomenon of interest only arises during a specified time period. It is possible to acquire frequencies and damping rates by using FT methods in principle, but this can be prohibitively expensive in simulations, may not provide useful information for transitory phenomena or is simply not possible to generate time traces of appropriate length. [2]

Signal processing methods from the toolbox can be helpful in solving these typical issues. These methods, which have been utilised in signal processing and structural mechanics for a long time but are not well-known to plasma simulation practitioners, contrast with the well-established Fourier transform. These methods, while more expensive than a basic

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Fourier transform, still provide a significant surplus value for a small cost. As a result, we're going to describe two methodologies that we've found beneficial in dealing with the two issues above in a pedagogical review: DMUSIC and stochastic system identification (SSI). However, compared to the expenses of gyrokinetic simulations, these methods necessitate the manipulation of large matrices and thus demand a lot more computational time .[3]

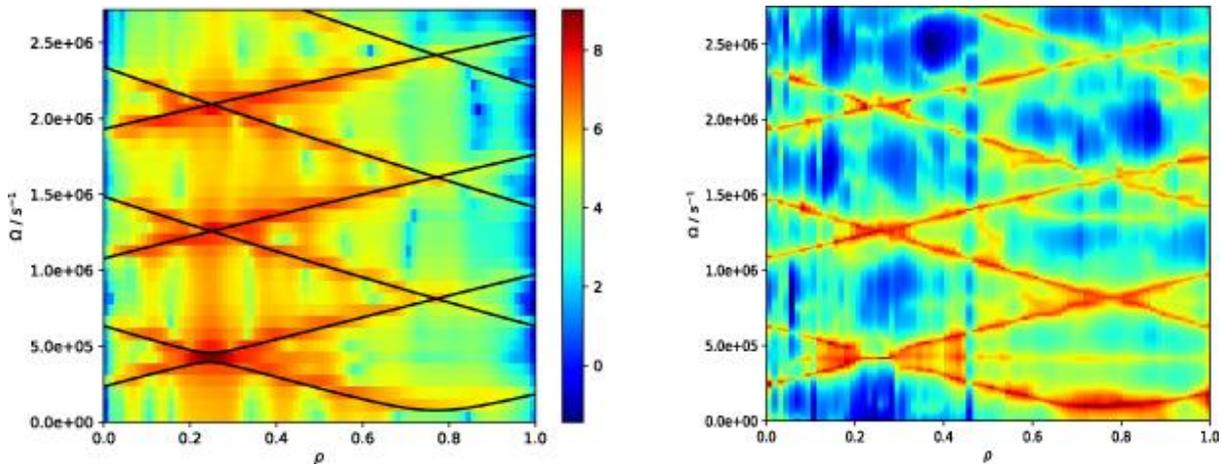
II. REVIEW OF LITERATURE

The topic of fusion energy research known as gyrokinetic validation involves comparing simulation results to experimental results using a rigorous technique (Terry et al.2008 [4]; Greenwald 2010 [5]). The drift-wave turbulence found in the core of fusion plasmas is well explained by the nonlinear gyrokinetic equations related to Maxwell's equations (Brizard & Hahm2007 [6]; Krommes 2010 [7]); nonlinear gyrokinetic theory is contained in numerical gyrokinetic codes (Garbet 2010 [8]). For many years, drift-wave turbulence in tokamak and stellarator plasmas has been widely studied (Horton 1999; Weiland 1999 [9]). When it comes to fusion plasmas, higher than previously recorded amounts of cross-field heat and particle movement drive this effort. For fusion plasmas with minor radii less than $0.0 < r/a < 0.9$, drift-wave instabilities and low frequency turbulence (with $\omega \ll \Omega_{ci}$) exhibit tiny amplitudes relative to background densities and temperatures (e.g. $n \sim 1\%$), and correlation lengths of $\sim 0.1 - 10 \rho_s$. Cyclotron frequency and sound radius are referred to here as Ω_{ci} and ρ_s , respectively. Electromagnetic disturbances, known as drift-wave instabilities, can occur at a range of normalised wavenumbers $0 < k\theta \rho_s < 60$, where $k\theta$ is the drift-wave poloidal wavenumber. The drift-wave turbulence described by nonlinear gyrokinetic theory leads to particle fluxes $\Gamma_j = \langle n_j v_{\perp} E_r \rangle + n_j \langle v_{\perp j} \| B \rangle / B$ and energy fluxes $Q_j = 3/2 \langle p_j v_{\perp} E_r \rangle + \langle q_{\perp j} \| B \rangle / B + 5/2 p_j \langle v_{\perp j} \| B \rangle / B$, where subscript j denotes the species (electrons or ions). The turbulent fluxes are affected by the density n , pressure p , parallel heat flux q_{\parallel} , parallel velocities v_{\parallel} , electric field E_r , and magnetic field B_r amplitude fluctuations. They also depend on these variables. On the other hand, the fluxes are also affected by their mean relative phases. Conway (2008) [10] states that the brackets signify ensemble averages of variable quantities.

Applications:

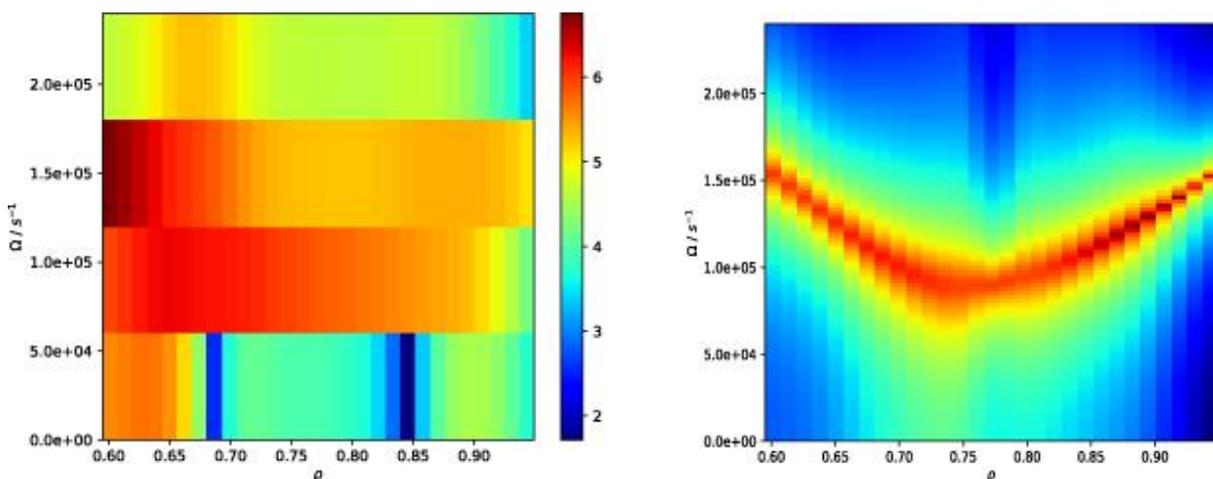
1. DMUSIC

Electromagnetic gyrokinetic simulations will use the two methodologies discussed above. Because these simulations are both numerically challenging and time consuming, the methods listed above are a logical choice. We will model Alfvén waves in tokamak equilibrium using the EUTERPE [11] three-dimensional global δf particle-in-cell code. A circular tokamak with an Aspect Ratio of $A = 10$ is the first example (the often-used ITPA standard case). Random noise is used to begin the simulation in order to ensure that all of the system's waves are excited. To guarantee that all waves in the system are excited, the simulation is started with random noise. 4×10^6 ion and 16×10^6 electron markers with a time step of $\Delta t = 5 \Omega_i^{-1}$ (here Ω_i denotes the ion gyro frequency) were evolved for a total of 6000 time steps (needing about 9 h on a Linux cluster with 48 processors). The radial resolution (using the normalised toroidal flux ρ as a radial coordinate) was $N_{\rho} = 80$ grid points and the simulation was restricted to $m = 5, \dots, 17$ and $n = -6$ toroidal and poloidal Fourier components, respectively. For further examination, the first 1500 time steps were deleted since they only show an initial numerical relaxation phase of the system.



DMUSIC and Fourier transforms to create an angular frequency spectrum with spatial resolution (**Fig. 1**; left; logarithm of Fourier amplitude modulus is shown) (right, the logarithm of P_r is shown). The ideal MHD continuum is depicted in the left panel by the solid lines. [12]

The CONTI code [13] was used to calculate the ideal magnetohydrodynamic (MHD) continuum, as shown by the solid line in the left panel. Low frequency toroidal Alfvén (TAE) gap at $\rho = 0.25$ is the only one. Tokamak equilibrium may not be circular, which could lead to voids appearing at the higher frequency branch crossings (e.g. ellipticity induced gaps). Due to the short time trace (4500 steps) the Fourier resolution is limited to $\Delta\Omega \approx 8 \times 10^4 \text{ s}^{-1}$ and nearly nothing can be concluded especially about the region $\rho > 0.6$. In particular, it is not possible to estimate the geodesic acoustic (GAM) up-shift of the lowest branch ($m = 11$) around $\rho = 0.8$. The result of zooming into this region can be seen in figure 2. (left).



In this **figure 2**, the same time series data are used to compute the GAM up-shifted branch from figure 1. DMUSIC on the left and the Fourier transform on the right.

As seen in **figure 1**, the same data can be processed using DMUSIC. We used 300×50 points to scan the relevant region of $s \in \mathbb{C}$ (the angular frequency Ω and damping factor, $\text{Im}(s)$ and $\text{Re}(s)$, respectively) to derive the signal function $P(s)$ for each ρ -position. $P_r(\Omega)$ is the reduced signal function that contains the signal's angular frequency component after it was integrated over the imaginary part of s .

2. SSI

In order to show the capabilities of SSI, a gyrokinetic simulation for a tokamak with $A = 3$ was used. The rotational-transform profile was chosen such that ideal MHD shows the presence of a reversed-shear Alfvén eigenmode (RSAE) together with a TAE. 16×10^6 ions and 64×10^6 electrons were used in the simulation resulting in a time series for the electrostatic potential consisting of 1.7×10^5 time steps with $\Delta t = 0.5 \Omega_i^{-1}$ (Ω_i denotes the ion gyro frequency), $N_\rho = 100$ radial grid points and eight ($m = 7, \dots, 14, n = -4$) Fourier components (the simulation ran for 56 h on 768 processors). To decrease the data quantity for the analysis, only every 320th time step was selected. The run time on one processor using a Julia version of the above approach is about 1 min what is small compared to the run duration of the simulation.

Due to their equal damping rates, both modes are still there at the end of the simulation, despite it taking a long time to run. The simulation data cannot be used to separate the modes, so SSI can be used to find their mode structures. The Hankel matrix would be enormous if SSI were applied to the time series of all 100×8 channels. A different approach was taken here in order to prevent this: each single Fourier component was used for a separate SSI analysis of order s resulting in frequencies $\lambda_{i,m}^s$ and corresponding $\bar{\lambda}_{i,m}^s$. The i, m pairs were then grouped together in such a way that within a group the pairwise differences between the real parts of $\lambda_{i,m}^s$, as well as that between the imaginary ones, are each smaller than given thresholds. The corresponding groups of $\lambda_{i,m}^s$ then constitute a mode. This approach is not one-size-fits-all: the grouping depends on the thresholds that must be chosen in an appropriate manner.

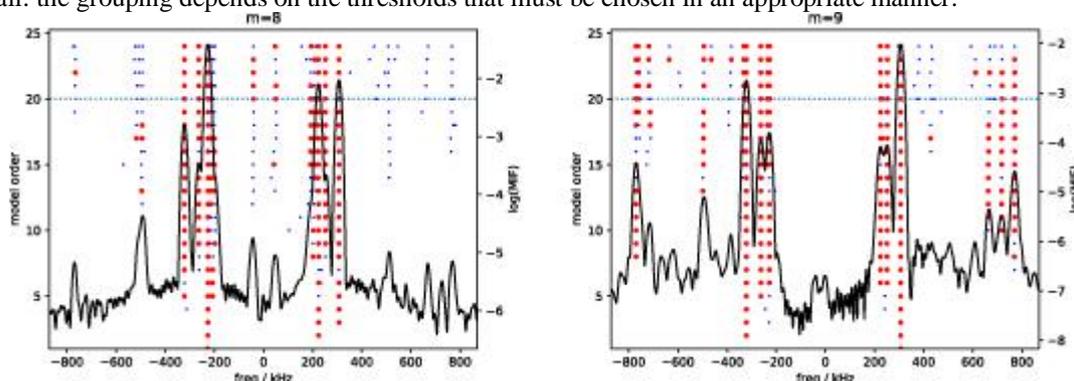


Figure 3. SSI stability diagram for $m = 8$ (left) and $m = 9$ (right) are shown (right). To distinguish between stable and unstable solutions, large red or small blue dots are placed next to them. This graph shows the logarithm of the Mode Indicator Function (MIF). Using the system order $s = 20$, we were able to rebuild the mode structure.

In addition to the four frequencies listed above, SSI is capable of detecting many others. Because they are either unstable (in this sense) or their amplitudes are modest, they have little impact on time series. A user's involvement is required to identify modes and frequencies with SSI, therefore it may be argued that the process isn't completely automated.

III. CONCLUSION

We applied these techniques to the findings of electromagnetic gyrokinetic simulations for tokamaks as an example. It is useful to have methods at our disposal that enable discovering frequencies, damping rates, and mode structures utilising shorter runs or that may be employed when it comes to analysing transient events because these simulations are quite expensive in terms of computation time. We have demonstrated how DMUSIC and SSI can be used to find the continuous Alfvén spectrum and identify TAE and RSAE mode structures.

When only a few modes predominate, such as in Alfvén mode simulations, SSI may be extremely helpful. Further research is needed to see if it can also be used to identify structures when there are multiple modes present, such as in turbulence.

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